Mr. Wright's Math Extravaganza

Precalculus
Additional Trigonometric Topics

Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: level 3.0 and success on applications

Score	I Can Statements
4.0	☐ I can demonstrate in-depth inferences and applications that go beyond what was taught.
3.5	In addition to score 3.0 performance, partial success at score 4.0 content.
3.0	☐ I can use the Law of Sines and Law of Cosines. ☐ I can use vectors. ☐ I can use the trigonometric form of a complex number.
2.5	No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content.
2.0	☐ I can find the area of a triangle. ☐ I can use the component form of vectors. ☐ I can find the magnitude and add, subtract, scalar multiply, and dot product vectors. ☐ I can find a unit vector. ☐ I can use vectors in trigonometric form. ☐ I can find the angle between two vectors. ☐ I can write complex numbers in trigonometric form. ☐ I can perform algebraic operations using complex numbers in trigonometric form.
1.5	Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content.
1.0	With help, partial success at score 2.0 content and score 3.0 content.
0.5	With help, partial success at score 2.0 content but not at score 3.0 content.
0.0	Even with help, no success.

Richard Wright

Precalculus

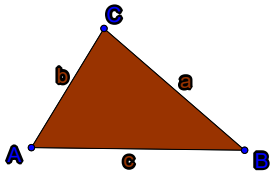
6-01 Law of Sines

Solve a triangle

- Find all _____and _____
- Use Law of Sines if you know
  - _____ (ASA or AAS)
  - _____ (SSA)

Law of Sines

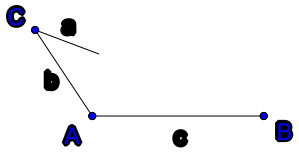
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



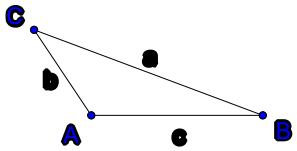
Solve ΔABC where $A = 30°$, $B = 45°$, and $a = 32$ ft

The Ambiguous Case (SSA)

- (Given A, a, b)
- If $A > 90°$ and
  - $a \le b$, then _____ solutions

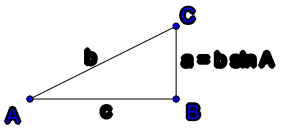


  - $a > b$, then _____ solution

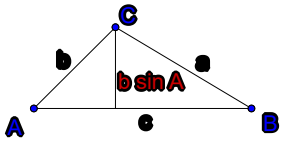


Precalculus 6-01 Name: _____

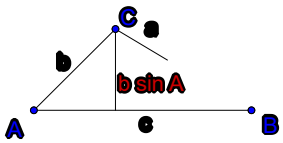
- If A < 90° and
  - $a = b \sin A$, then _____ solution



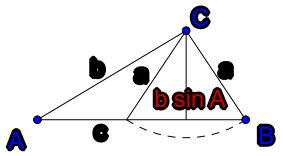
  - $a > b \sin A$ and $a \geq b$, _____ solution



  - $a < b \sin A$, then _____ solutions



  - $b \sin A < a < b$, then _____ solutions



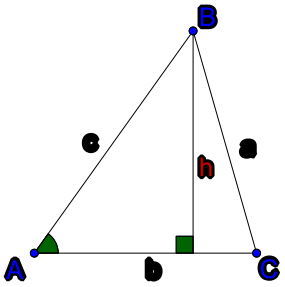
Solve ΔABC where $A = 58°$, $a = 4.5$, and $b = 5$

Area of a Triangle

$$Area = \frac{1}{2}bc \sin A$$

$$Area = \frac{1}{2}ac \sin B$$

$$Area = \frac{1}{2}ab \sin C$$



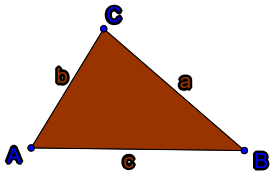
Precalculus

6-02 Law of Cosines

- When you _____use _____
- _____

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



Solve ΔABC where a = 20, b = 18, c = 13

Area of a Triangle given all Sides

_____ Formula

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

- Where $s = \frac{a+b+c}{2}$

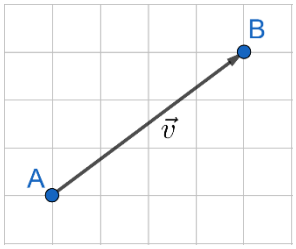
Find the area of a triangle with sides 14 cm, 21 cm, 27 cm

Precalculus

6-03 Vectors

Vector

- _____line segment $\vec{v}$
- Has _____and _____
- Magnitude $\|\vec{v}\|$ is _____of the segment



Component form

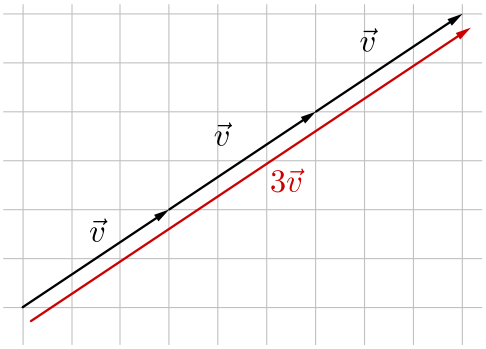
- $\vec{v} = \langle v_1, v_2 \rangle$
- _____– _____point
- $\vec{v} = \langle q_1 - p_1,\ q_2 - p_2 \rangle = \langle v_1, v_2 \rangle$
- $\|\vec{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2}$
  $= \sqrt{v_1^2 + v_2^2}$

Find the component form of the vector and its magnitude if its initial point is (1, 7) and its terminal point is (4, 3).

Vector Operations

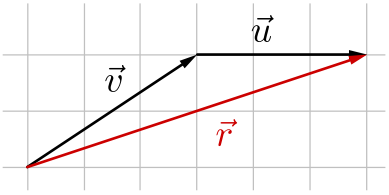
Scalar Multiplication

- $k\,\vec{v} = \langle kv_1, kv_2 \rangle$
- If k is negative it goes in _____direction



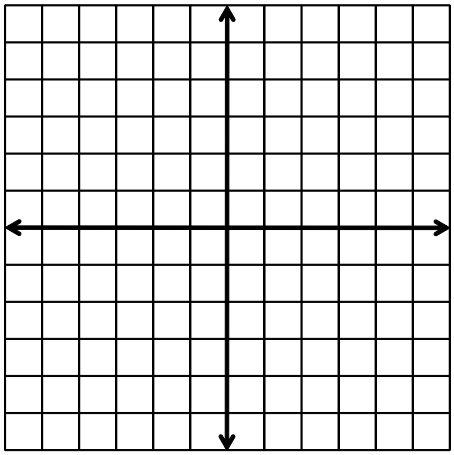
Add

- Add _____components
- $\vec{v} + \vec{u} = \langle v_1 + u_1,\ v_2 + u_2 \rangle$

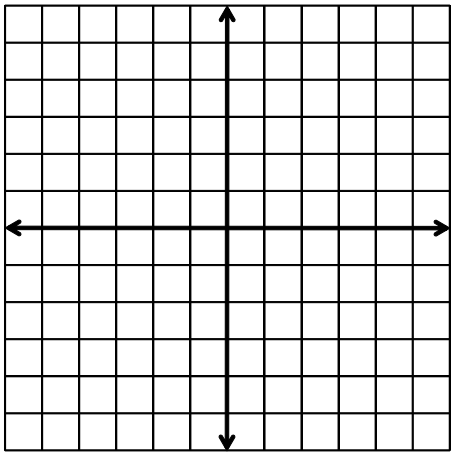


$\langle 2,\ 3 \rangle + \langle 1,\ 0 \rangle$

Let $\vec{u} = \langle 1,\ 6\rangle$ and $\vec{v} = \langle -4,\ 2\rangle$, find $3\vec{u}$



Let $\vec{u} = \langle 1,\ 6\rangle$ and $\vec{v} = \langle -4,\ 2\rangle$, find $2\vec{v} + \vec{u}$



Unit Vectors

- Vector in the _____direction, but magnitude is _____
  - $\hat{u} = \frac{\vec{v}}{\|\vec{v}\|}$
- Special Unit Vectors
  - $\hat{\imath}$ =_____
  - $\hat{\jmath}$ =_____

Linear Combination Form

- $3\hat{\imath} + 2\hat{\jmath} = \langle 3,\ 2\rangle$

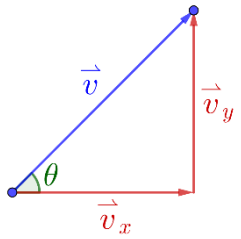
Let $\vec{v} = 3\hat{\imath} - 4\hat{\jmath}$ and $\vec{w} = 2\hat{\imath} + 9\hat{\jmath}$, find $2\vec{v} + \vec{w}$.

Precalculus

6-04 Writing Vectors in Trigonometric Form

Direction Angle

- $v_x = \|\vec{v}\| \cos\theta$
- $v_y = \|\vec{v}\| \sin\theta$
- $\vec{v} = \|\vec{v}\|\langle \cos\theta\,, \sin\theta \rangle$
- $\tan\theta = \frac{v_y}{v_x}$



Write the vector in trig form. $\langle -12,\ 5 \rangle$

Write the vector in component form. $10\langle \cos 120°\,, \sin 120° \rangle$

Find the component form of the vector representing velocity of an airplane descending at 100 mph at 45° below the horizontal.

Add the vectors. Write the result in trig form. $4\langle \cos 210°\,, \sin 210° \rangle + 2\langle \cos 30°\,, \sin 30° \rangle$

An airplane is traveling at 724 km/h at 30° E of N. If the wind velocity is 32 km/h from the west, find the resultant speed and direction of the plane.

Precalculus

6-05 Dot Products

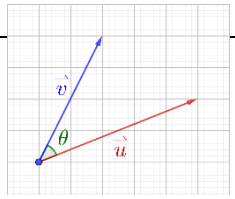
Dot Product

- $\vec{u} = \langle u_1, u_2 \rangle, \vec{v} = \langle v_1, v_2 \rangle$
- $\vec{u} \bullet \vec{v} = u_1 v_1 + u_2 v_2$

Find $\langle 5, -4 \rangle \bullet \langle 9, -2 \rangle$

Angle between vectors

- $\vec{u} \bullet \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$



Find the angle between $\langle 5, -4 \rangle$ and $\langle 9, -2 \rangle$

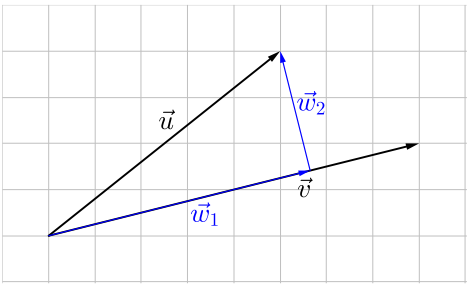
Parallel and Perpendicular Vectors

- If $\vec{u} \bullet \vec{v} = 0$, then $\vec{u}$ and $\vec{v}$ are _____(perpendicular)
- If $\vec{u} = k\vec{v}$, then $\vec{u}$ and $\vec{v}$ are _____(or antiparallel)

Are $\langle 1, -4 \rangle$ and $\langle 6, 2 \rangle$ orthogonal, parallel, or neither?

Find Vector Components

- Let $\vec{u}$ and $\vec{v}$ be vectors such that $\vec{u} = \overrightarrow{w_1} + \overrightarrow{w_2}$ where $\overrightarrow{w_1}$ and $\overrightarrow{w_2}$ are orthogonal and $\overrightarrow{w_1}$ is parallel to $\vec{v}$. $\overrightarrow{w_1}$ and $\overrightarrow{w_2}$ are components of $\vec{u}$.
- $\overrightarrow{w_1}$ is the projection of $\vec{u}$ onto $\vec{v}$: $\overrightarrow{w_1} = proj_{\vec{v}}\vec{u}$
- $\overrightarrow{w_1} = proj_{\vec{v}}\vec{u} = \frac{\vec{u}\bullet\vec{v}}{\|\vec{v}\|^2}\vec{v}$
- $\overrightarrow{w_2} = \vec{u} - \overrightarrow{w_1}$
- $Work = \vec{F} \bullet \vec{d}$



Find the projection of $\vec{u} = \langle 3,\ 4\rangle$ onto $\vec{v} = \langle 8,\ 2\rangle$. Then write $\vec{u}$ as the sum of 2 orthogonal vectors.

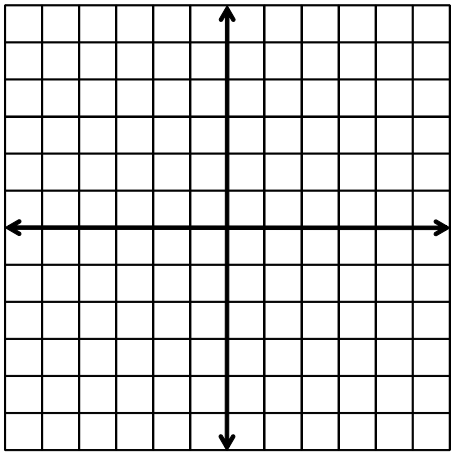
Precalculus

6-06 Trigonometric Form of a Complex Number

Graph Complex Number

- $a + bi$
- Graph by moving _____a, and _____b
- x-axis is _____
- y-axis is _____

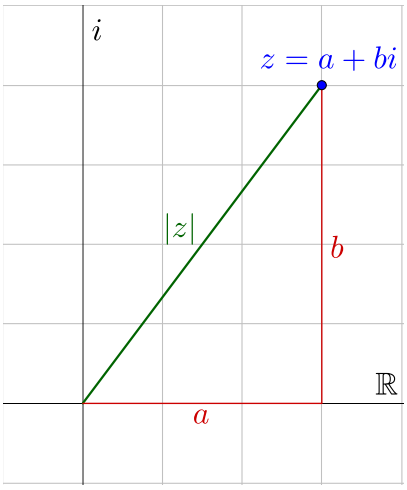
Graph (a) $2 + 3i$ and (b) $-3 - 4i$



Absolute Value of a Complex Number

- Absolute value is _____from _____
  - $|a + bi| = \sqrt{a^2 + b^2}$

$|4 + i|$



Trig Form of a Complex Number

- $a = r\cos\theta$
- $b = r\sin\theta$
- $r = \sqrt{a^2 + b^2}$
- $\tan\theta = \frac{b}{a}$

- $z = a + bi$
- $z = r\cos\theta + r\sin\theta\ i$
- $z = r(\cos\theta + i\sin\theta)$
  - r is _____, θ is _____

Write in standard form: $z = 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

Write in trig form: $z = -2 - 2i$

Precalculus

6-07 Trigonometric Form of a Complex Number Operations

Multiplication and Division

- If $z_1 = r_1(\cos\theta + i\sin\theta)$ and $z_2 = r_2(\cos\theta + i\sin\theta)$, then

$$z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2))$$

If $z_1 = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ and $z_2 = 6\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$, find

$z_1 z_2$ $\frac{z_1}{z_2}$

Exponents

$$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$$

Let $z = 1 + i$, find z^4

Roots of Complex Numbers

$$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)\right)$$

- Where k = 0, 1, 2, …, n − 1
- These are _____out evenly around a circle with _____$\sqrt[n]{r}$

Find the cube roots of $-6 + 6i$