

Precalculus

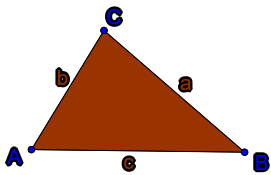
6-01 Law of Sines

Solve a triangle

- Find all _____ and _____
- Use Law of Sines if you know
 - _____ (ASA or AAS)
 - _____ (SSA)

Law of Sines

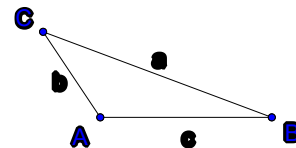
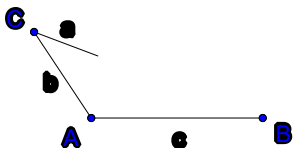
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Solve $\triangle ABC$ where $A = 30^\circ$, $B = 45^\circ$, and $a = 32$ ft

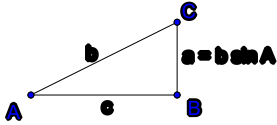
The Ambiguous Case (SSA)

- (Given A, a, b)
- If $A > 90^\circ$ and
 - $a \leq b$, then _____ solutions
 - $a > b$, then _____ solution

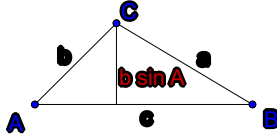


- If $A < 90^\circ$ and

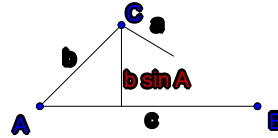
- $a = b \sin A$, then _____ solution



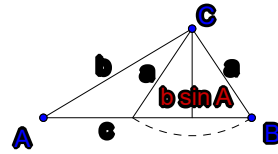
- $a > b \sin A$ and $a \geq b$, _____ solution



- $a < b \sin A$, then _____ solutions



- $b \sin A < a < b$, then _____ solutions



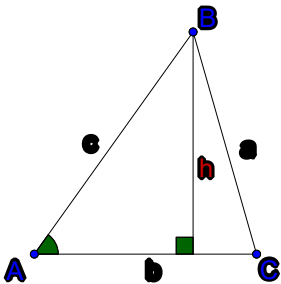
Solve $\triangle ABC$ where $A = 58^\circ$, $a = 4.5$, and $b = 5$

Area of a Triangle

$$Area = \frac{1}{2} bc \sin A$$

$$Area = \frac{1}{2} ac \sin B$$

$$Area = \frac{1}{2} ab \sin C$$



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6-02 Law of Cosines

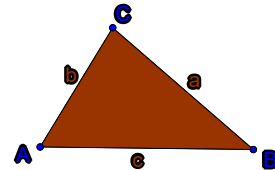
- When you _____ use _____
- _____

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Solve $\triangle ABC$ where $a = 20$, $b = 18$, $c = 13$

Area of a Triangle given all Sides

_____ Formula

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

- Where $s = \frac{a+b+c}{2}$

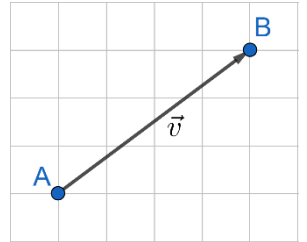
Find the area of a triangle with sides 14 cm, 21 cm, 27 cm

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6-03 Vectors

Vector

- _____ line segment \vec{v}
- Has _____ and _____
- Magnitude $\|\vec{v}\|$ is _____ of the segment



Component form

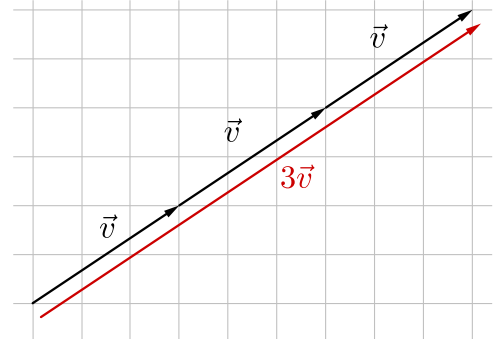
- $\vec{v} = \langle v_1, v_2 \rangle$
- _____ - _____ point
- $\vec{v} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle$
- $\|\vec{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2}$
 $= \sqrt{v_1^2 + v_2^2}$

Find the component form of the vector and its magnitude if its initial point is (1, 7) and its terminal point is (4, 3).

Vector Operations

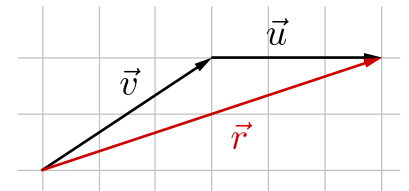
Scalar Multiplication

- $k\vec{v} = \langle kv_1, kv_2 \rangle$
- If k is negative it goes in _____ direction



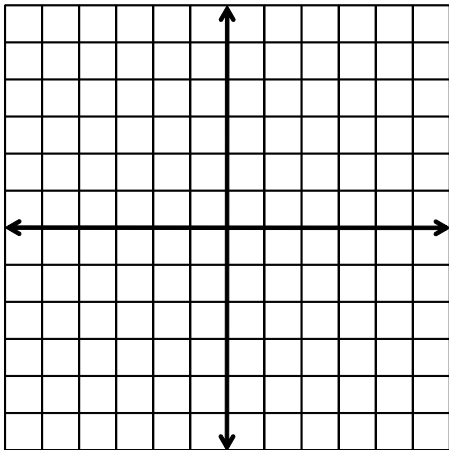
Add

- Add _____ components
- $\vec{v} + \vec{u} = \langle v_1 + u_1, v_2 + u_2 \rangle$

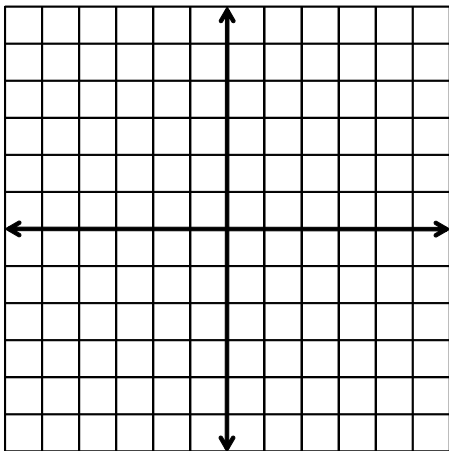


$\langle 2, 3 \rangle + \langle 1, 0 \rangle$

Let $\vec{u} = \langle 1, 6 \rangle$ and $\vec{v} = \langle -4, 2 \rangle$, find $3\vec{u}$



Let $\vec{u} = \langle 1, 6 \rangle$ and $\vec{v} = \langle -4, 2 \rangle$, find $2\vec{v} + \vec{u}$



Unit Vectors

- Vector in the _____ direction, but magnitude is _____
 - $\hat{u} = \frac{\vec{v}}{\|\vec{v}\|}$
- Special Unit Vectors
 - $\hat{i} =$ _____
 - $\hat{j} =$ _____

Linear Combination Form

- $3\hat{i} + 2\hat{j} = \langle 3, 2 \rangle$

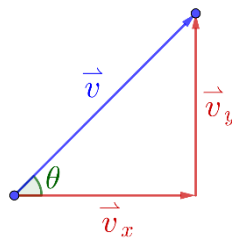
Let $\vec{v} = 3\hat{i} - 4\hat{j}$ and $\vec{w} = 2\hat{i} + 9\hat{j}$, find $2\vec{v} + \vec{w}$.

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6-04 Writing Vectors in Trigonometric Form

Direction Angle

- $v_x = \|\vec{v}\| \cos \theta$
- $v_y = \|\vec{v}\| \sin \theta$
- $\vec{v} = \|\vec{v}\| \langle \cos \theta, \sin \theta \rangle$
- $\tan \theta = \frac{v_y}{v_x}$



Write the vector in trig form. $\langle -12, 5 \rangle$

Write the vector in component form. $10 \langle \cos 120^\circ, \sin 120^\circ \rangle$

Find the component form of the vector representing velocity of an airplane descending at 100 mph at 45° below the horizontal.

Add the vectors. Write the result in trig form. $4 \langle \cos 210^\circ, \sin 210^\circ \rangle + 2 \langle \cos 30^\circ, \sin 30^\circ \rangle$

An airplane is traveling at 724 km/h at 30° E of N. If the wind velocity is 32 km/h from the west, find the resultant speed and direction of the plane.

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6-05 Dot Products

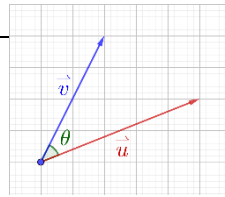
Dot Product

- $\vec{u} = \langle u_1, u_2 \rangle, \vec{v} = \langle v_1, v_2 \rangle$
- $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2$

Find $\langle 5, -4 \rangle \cdot \langle 9, -2 \rangle$

Angle between vectors

- $\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta$



Find the angle between $\langle 5, -4 \rangle$ and $\langle 9, -2 \rangle$

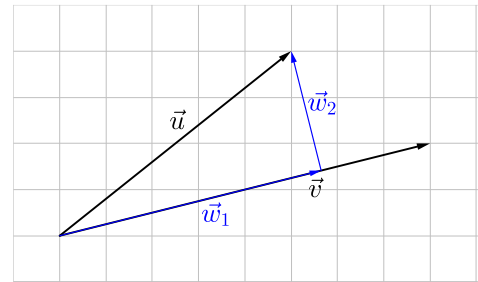
Parallel and Perpendicular Vectors

- If $\vec{u} \cdot \vec{v} = 0$, then \vec{u} and \vec{v} are _____ (perpendicular)
- If $\vec{u} = k\vec{v}$, then \vec{u} and \vec{v} are _____ (or antiparallel)

Are $\langle 1, -4 \rangle$ and $\langle 6, 2 \rangle$ orthogonal, parallel, or neither?

Find Vector Components

- Let \vec{u} and \vec{v} be vectors such that $\vec{u} = \vec{w}_1 + \vec{w}_2$ where \vec{w}_1 and \vec{w}_2 are orthogonal and \vec{w}_1 is parallel to \vec{v} . \vec{w}_1 and \vec{w}_2 are components of \vec{u} .
- \vec{w}_1 is the projection of \vec{u} onto \vec{v} : $\vec{w}_1 = \text{proj}_{\vec{v}} \vec{u}$
- $\vec{w}_1 = \text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v}$
- $\vec{w}_2 = \vec{u} - \vec{w}_1$
- $\text{Work} = \vec{F} \cdot \vec{d}$



Find the projection of $\vec{u} = \langle 3, 4 \rangle$ onto $\vec{v} = \langle 8, 2 \rangle$. Then write \vec{u} as the sum of 2 orthogonal vectors.

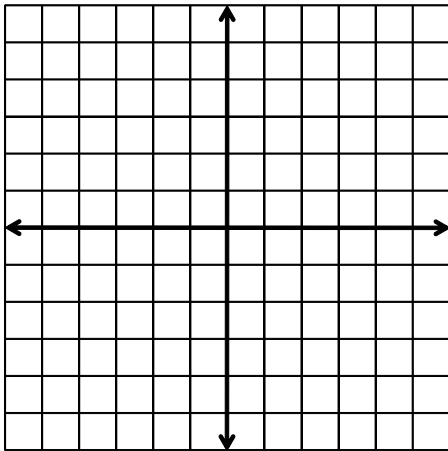
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6-06 Trigonometric Form of a Complex Number

Graph Complex Number

- $a + bi$
- Graph by moving _____ a , and _____ b
- x -axis is _____
- y -axis is _____

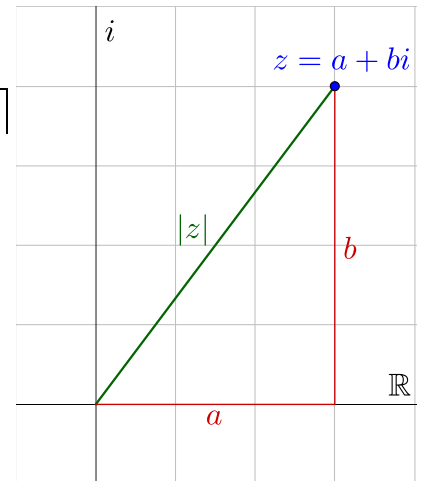
Graph (a) $2 + 3i$ and (b) $-3 - 4i$



Absolute Value of a Complex Number

- Absolute value is _____ from _____
 - $|a + bi| = \sqrt{a^2 + b^2}$

$|4 + i|$



Trig Form of a Complex Number

- $a = r \cos \theta$
- $b = r \sin \theta$
- $r = \sqrt{a^2 + b^2}$
- $\tan \theta = \frac{b}{a}$
- $z = a + bi$
- $z = r \cos \theta + r \sin \theta i$
- $z = r(\cos \theta + i \sin \theta)$
 - r is _____, θ is _____

Write in standard form: $z = 8 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$

Write in trig form: $z = -2 - 2i$

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6-07 Trigonometric Form of a Complex Number Operations

Multiplication and Division

- If $z_1 = r_1(\cos \theta + i \sin \theta)$ and $z_2 = r_2(\cos \theta + i \sin \theta)$, then

$$z_1 z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} (\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2))$$

If $z_1 = 3 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$ and $z_2 = 6 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$, find

$z_1 z_2$

$\frac{z_1}{z_2}$

z_2

Exponents

$$z^n = r^n (\cos(n\theta) + i \sin(n\theta))$$

Let $z = 1 + i$, find z^4

Roots of Complex Numbers

$$\sqrt[n]{z} = \sqrt[n]{r} \left(\cos \left(\frac{\theta}{n} + \frac{2\pi k}{n} \right) + i \sin \left(\frac{\theta}{n} + \frac{2\pi k}{n} \right) \right)$$

- Where $k = 0, 1, 2, \dots, n - 1$
- These are _____ out evenly around a circle with _____ $\sqrt[n]{r}$

Find the cube roots of $-6 + 6i$